

# SH

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$$u_t = \otimes u + \otimes f(u) \quad ( )$$

$\otimes f(u) \otimes u$

$\otimes \otimes$

t

u

u\_t

f(u)

$$f(t) = f_j(t) + \sum_{k=j}^{+\infty} d_k(t) \quad (1)$$

Semi group

[ ]

SH

$$( \quad ) L^2(\mathbb{R})$$

$$\int_{-\infty}^{+\infty} |f(t)|^2 dt < +\infty$$

$$v_j \{w_k\}$$

$$f(t)$$

$$f_j(t) \quad v_j \quad j$$

$$( \quad )$$

$$k = j, \dots, \infty \quad w_k \quad d_k(t)$$

)

(

$$j$$

$$2^{-j}$$

$$v_j$$

$$( \quad )$$

)

$$2^{-j} ($$

$$v_j$$

$$v_j$$

$$: ( \quad )$$

$$f_j(t) \in v_j \Leftrightarrow f_{j+1}(t) \in v_{j+1} \quad v_j \subset v_{j+1}; j \in \mathbb{Z}$$

$$( \quad ) \quad f_j(t) \quad f_{j+1}(t) \quad j=0 \quad ( \quad )$$

$$d_j(t) = f_{j+1}(t) - f_j(t); \quad d_j(t) \in w_j \quad (1) \quad 1/2^j \quad j$$

$$v_{j+1} = v_j \oplus w_j \quad (2) \quad j \quad f(t)$$

$$j \quad w_j \quad v_j \quad d_j(t) \quad j+1 \quad f_j(t)$$

$$\varphi(x-k) \in v_0, \psi(x-k') \in w_0 \Rightarrow$$

$$\int \varphi(x-k) * \psi(x-k') dx = 0$$

$$f_{j+1}(t) = f_j(t) + d_j(t) \quad (3)$$

$$\begin{aligned} \varphi(t) &\in v_0 & ( ) \\ \varphi(t) & & v_1 \quad \varphi(t) \\ & & \vdots \\ & & \varphi(2t) \end{aligned}$$

$$\varphi(t) = \sum_n h(n) \sqrt{2} \cdot \phi(2t - n), n \in Z \quad ( )$$

$$\sqrt{2} \quad h(n) \quad ( )$$

$$( )$$

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-

$$\begin{aligned} &w_j & v_j & v_{j+1} \\ ( ) & & & \\ & & & \\ & & & \\ & \cdot & v_0 & v_1 & \cdot & w_0 \end{aligned}$$

$$\begin{aligned} \psi_k(t) &= \psi(t-k) & w_0 \\ & \vdots & v_1 & w_0 \end{aligned}$$

$$\psi(t) = \sum h_1(n) \sqrt{2} \cdot \varphi(2t - n) \quad n \in Z \quad ( )$$

$$\vdots \quad w_j \quad v_j$$

$$\langle \varphi_{j,k}(t), \psi_{j,k}(t) \rangle = \int \varphi_{j,k}(t) \psi_{j,k}(t) dt = 0 \quad ( )$$

$j, k, l \in Z$

$$\begin{aligned} &h(n) & h_1(n) \\ & & \vdots \\ & & \end{aligned}$$

$$h_1(n) = (-1)^n h(N-1-n) \quad ( )$$

$\psi(t)$

$$\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k) \quad ( )$$

[ ] Daubechies

$$\begin{aligned} v_{j+1} &= w_j \oplus w_{j-1} \oplus v_{j-1} = \dots = \\ &w_j \oplus w_{j-1} \oplus w_{j-2} \oplus \dots \oplus w_{j-j} \oplus v_{j-j} \end{aligned} \quad ( )$$

$$\lim_{j \rightarrow \infty} v_j = \{0\} \quad ( )$$

$$\lim_{j \rightarrow \infty} v_j = L^2(R) \quad ( )$$

$$f(t) \in v_j \Leftrightarrow f(2^*t) \in v_{j+1} \quad ( )$$

$$f(t) \in v_j \Leftrightarrow f(t-k) \in v_j \quad ( )$$

$$(\ ) \varphi$$

$$\begin{aligned} &v_0 \\ & \vdots \\ & v_0 \end{aligned} \quad \begin{aligned} &v_0 \\ & \vdots \\ & v_0 \end{aligned}$$

$\therefore v_0 = \overline{\text{span}\{\varphi_k(t)\}_k} \quad \varphi_k(t) = \varphi(t-k) \quad ( )$

$$f(t) = \sum_k a_k \cdot \varphi_k(t) \quad f(t) \in v_0 \text{ for } \quad ( )$$

$v_j \quad \varphi_{j,k}(t)$

$$v_j = \overline{\text{span}\{\varphi_{j,k}(t)\}_k} \quad ( )$$

$$\varphi_{j,k}(t) = 2^{j/2} \varphi(2^j t - k) \quad ( )$$

$$\text{if } f(t) \in v_j \Rightarrow f(t) = \sum_k a_k \varphi(2^j t - k) \quad ( )$$

$$\begin{aligned} &2^j & \varphi_{j,k}(t) \\ & & k \times 2^{-j} \\ & \varphi_{j,k}(t) & j > 0 \end{aligned}$$

$$\varphi_k(t) \quad \varphi_{j,k}(t)$$

$$j < 0$$

$$( ) \quad v_j$$

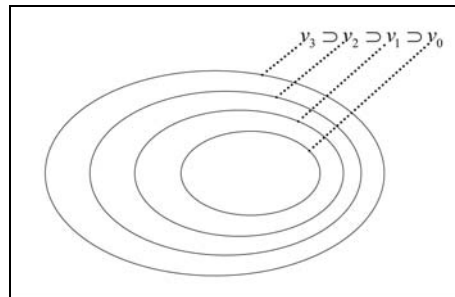
$$c(k) = c_0(k) = \langle g(t), \varphi_k(t) \rangle = \int g(t) \cdot \varphi_k(t) dt \quad ( )$$

$$d_j(k) = d(j, k) = \langle g(t), \psi_{j,k}(t) \rangle = \int g(t) \cdot \psi_{j,k}(t) dt \quad ( )$$

$$g_0 = \sum_{k=-\infty}^{+\infty} c(k) \cdot \varphi_k(t) \quad ) L^2(\mathbb{R})$$

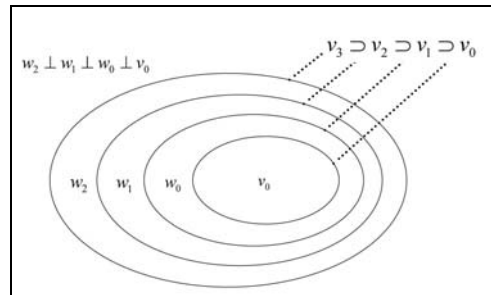
$$v_0 \quad g \quad :$$

$$v_0 \quad \sum_{k=-\infty}^{+\infty} d(0, k) \cdot \psi_{0,k}(t) \quad g(t) = \sum_{k=-\infty}^{+\infty} c(k) \cdot \varphi_k(t) + \sum_{j=0}^{+\infty} \sum_{k=-\infty}^{+\infty} d(j, k) \cdot \psi_{j,k}(t) \quad ( )$$

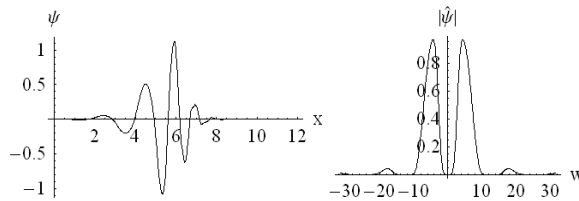
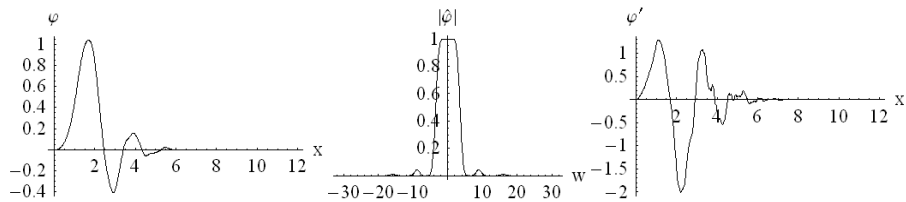


$v_i; i = 0, 1, 2, 3, \dots$

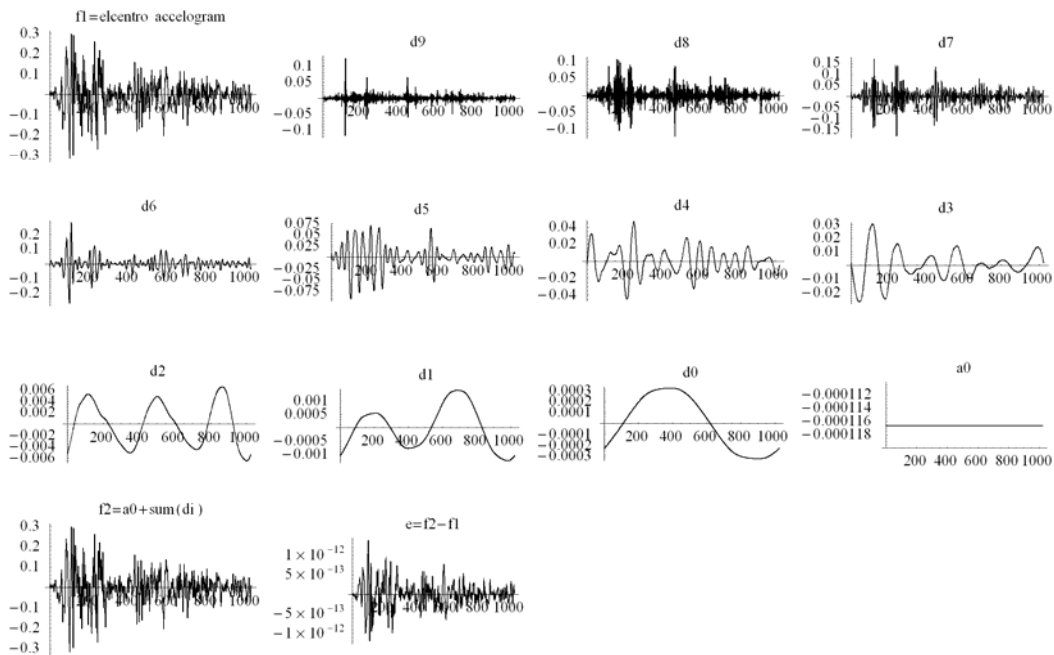
$\cdot [ ] \quad \varphi_{i,k}$



$\cdot [ ] \quad v_{i+1} \quad w_i \quad v_i$



**Db**



**Db**

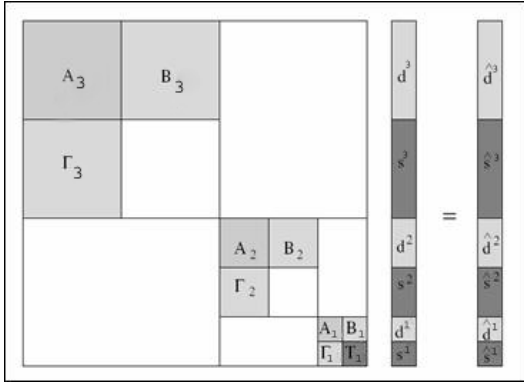
$$\begin{array}{r}
 \begin{pmatrix} w_j, v_j \\ \vdots \\ v_J \end{pmatrix} \begin{pmatrix} T \\ \vdots \\ g \end{pmatrix} = \begin{pmatrix} \hat{d}^j, \hat{s}^j \\ \vdots \\ \{d^j, s_0\} \end{pmatrix} \\
 \left. \begin{array}{l} j=0,1,\dots,n-1 \\ \vdots \\ j \end{array} \right\} \\
 \left. \begin{array}{l} \vdots \\ w_j \quad v_j \\ \vdots \\ P_j : L^2(R) \rightarrow v_j \end{array} \right\} \\
 \left( \begin{array}{c} a_0 \\ \vdots \\ a_0 \quad d_9 \quad d_0 \\ \vdots \\ v_9 \end{array} \right) \\
 A_j = Q_j T Q_j ; \quad \left( \right) \\
 A_j : w_j \rightarrow w_j ; \\
 B_j : v_j \rightarrow w_j ; \quad B_j = Q_j T P_j ; \\
 \Gamma_j = P_j T Q_j \quad \Gamma_j : w_j \rightarrow v_j ; \\
 T_j = P_j T P_j \quad T_j : v_j \rightarrow v_j ; \\
 \vdots \quad v_j \\
 P_j : L^2(R) \rightarrow v_j \\
 (P_j f)(x) = \sum_k \langle f, \phi_{j,k} \rangle \phi_{j,k}(x) \\
 \vdots \quad w_j \\
 Q_j = P_{j+1} - P_j \quad Q_j : L^2(R) \rightarrow w_j ; \\
 \left( \right) \quad \left( \right) \quad T \\
 T : L^2(R) \rightarrow L^2(R) \\
 \text{(NS form)} \\
 \left\{ A_j, B_j, \Gamma_j \right\}
 \end{array}$$

$$s_l = \int_{-\infty}^{+\infty} \varphi(x-l) \frac{d}{dx} \varphi(x) dx$$

$$\gamma_l = \int_{-\infty}^{+\infty} \varphi(x-l) \frac{d}{dx} \psi(x) dx;$$

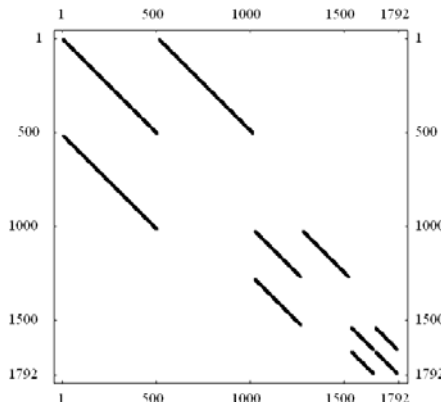
( )

$$0 \quad \left\{ \hat{d}^j, \hat{s}^j \right\} \quad j = 0, 1, \dots, n-1 \quad \left\{ \left\{ d^j \right\}, s_0 \right\}$$



$$\left\{ \left\{ \hat{d}^j \right\}, \left\{ \hat{s}^j \right\} \right\} \quad \left\{ \left\{ d^j \right\}, \left\{ s^j \right\} \right\}$$

[ ]  $w_j \quad v_j$



$d / dx$

Db12

semi group

[ ] PDEs

T

:

$$T = \left\{ A_j, B_j, \Gamma_j \right\}_{j \in \mathbb{Z}} \quad ( )$$

:

$2^n$

$$T = \left\{ \left\{ A_j, B_j, \Gamma_j \right\}_{j \in \mathbb{Z}; j \leq n-1}, T_0 \right\} \quad ( )$$

$$T_0 = P_0 T P_0 \quad ( )$$

$$T = \sum_{j=1}^{+\infty} (Q_j T Q_j + Q_j T P_j + P_j T P_j) + P_0 T P_0 \quad ( )$$

(NS )

$$d^i, s^i \quad ( )$$

( )  $NS \quad \hat{d}^i, \hat{s}^i$

$d / dx$

Db12

[ ]

$A_j, B_j, \Gamma_j, T_j$

$\frac{d}{dx}$

:

$\alpha^j, \beta^j, \gamma^j, s^j$

$$\alpha_{il}^j = 2^j \int_{-\infty}^{+\infty} \psi(2^j x - i) \psi'(2^j x - l) \cdot 2^j dx = 2^j \alpha_{i-l}^j$$

$$\beta_{il}^j = 2^j \int_{-\infty}^{+\infty} \psi(2^j x - i) \varphi'(2^j x - l) \cdot 2^j dx = 2^j \beta_{i-l}^j$$

( )

$$\gamma_{il}^j = 2^j \int_{-\infty}^{+\infty} \varphi(2^j x - i) \psi'(2^j x - l) \cdot 2^j dx = 2^j \gamma_{i-l}^j$$

$$s_{il}^j = 2^j \int_{-\infty}^{+\infty} \varphi(2^j x - i) \varphi'(2^j x - l) \cdot 2^j dx = 2^j s_{i-l}^j$$

$$\beta_l = \int_{-\infty}^{+\infty} \psi(x-l) \frac{d}{dx} \varphi(x) dx$$

$$\alpha_l = \int_{-\infty}^{+\infty} \psi(x-l) \frac{d}{dx} \psi(x) dx;$$

$$Q_0(\otimes \Delta t) = e^{\otimes \Delta t}$$

( )

$$Q_1(\otimes \Delta t) = (e^{\otimes \Delta t} - \text{Ⓢ})(\otimes \Delta t)^{-1}$$

$$Q_2(\otimes \Delta t) = (e^{\otimes \Delta t} - \text{Ⓢ} - \otimes \Delta t)(\otimes \Delta t)^{-2}$$

**SH**

(x, z)

$$f_y(x, z), \mu(x, z), \rho(x, z), v_y(x, z), u_y(x, z)$$

**SH**

$$\rho \frac{\partial^2 u_y}{\partial t^2} = \frac{\partial}{\partial x} \left( \mu \frac{\partial u_y}{\partial x} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial u_y}{\partial z} \right) + f_y \quad ( )$$

Ⓢ<sub>y</sub>

$$\frac{\partial^2 u_y}{\partial t^2} = \otimes_y + \frac{f_y}{\rho} \quad ( )$$

$$\otimes_y = \frac{1}{\rho} \frac{\partial}{\partial x} \left( \mu \frac{\partial u_y}{\partial x} \right) + \frac{1}{\rho} \frac{\partial}{\partial z} \left( \mu \frac{\partial u_y}{\partial z} \right) \quad ( )$$

semi group

$$\frac{\partial v_y}{\partial t} = \otimes_y u_y + \frac{f_y}{\rho}, \quad \frac{\partial u_y}{\partial t} = v_y \quad ( )$$

$$\mathbf{U} = \mathbf{L}\mathbf{U} + \mathbf{F} \quad ( )$$

$$\mathbf{U} = \begin{pmatrix} u_y \\ v_y \end{pmatrix}; \mathbf{L} = \begin{pmatrix} 0 & \mathbf{I} \\ \otimes_y & 0 \end{pmatrix}; \mathbf{F} = \frac{1}{\rho} \begin{pmatrix} 0 \\ f_y \end{pmatrix} \quad ( )$$

$$\mathbf{U}_{n+1} = e^{\Delta t \mathbf{L}} \mathbf{U}_n + \Delta t \cdot \beta_0 \cdot \mathbf{F}_n \quad \gamma = 0 \ \& \ M = 1 \Rightarrow \quad ( )$$

γ = 0, l = 1

M	β <sub>0</sub>	β <sub>1</sub>	β <sub>2</sub>	order
	Q <sub>1</sub>	0	0	
	Q <sub>1</sub> + Q <sub>2</sub>	-Q <sub>2</sub>	0	
	Q <sub>1</sub> + 3Q <sub>2</sub> / 2 + Q <sub>3</sub>	-2(Q <sub>2</sub> + Q <sub>3</sub> )	Q <sub>2</sub> / 2 + Q <sub>3</sub>	

$$\ln u_{,t} = \otimes u + \text{Ⓢ} f(u) \quad \Omega \in R^d \quad ( )$$

$$\ln t \in [0, T], \partial \Omega \in R^{d-1} \quad \text{On } \text{Ⓢ} u(x, t) = 0 \ \Omega;$$

$$u(x, 0) = u_0(x)$$

Ⓢ<sub>Ⓢ</sub> ⊗

$$u(x, 0) \quad Bu(x, t)$$

semi group

$$u(x, t) = e^{t \otimes} u(x, 0) + \int_0^t e^{(t-\tau) \otimes} \text{Ⓢ} (u(x, \tau)) d\tau \quad ( )$$

Ⓢ

u(x, t)

$$u_n = u(x, t_n) \quad \Delta t \quad t_n = t_0 + n \Delta t$$

$$N_n \equiv \text{Ⓢ} (u(x, t_n))$$

( )

$$u_{n+1} = e^{q \cdot l \Delta t} u_{n+1-l} + \Delta t (\gamma \cdot N_{n+1} + \sum_{m=0}^{M-1} \beta_m \cdot N_{n-m}) \quad ( )$$

M + 1

$$q \cdot \Delta t \quad \beta_m \quad \gamma \quad l \leq M$$

γ = 0

$$\gamma = 0, l = 1 \quad ( )$$

$$\beta_n \quad \gamma \quad M = 1, 2, 3$$

( ) ( )

$$Q_j(x) = \frac{e^x - E_j(x)}{x^j} \quad ( )$$

$$E_j(x) = \sum_{k=0}^{j-1} \frac{x^k}{k!} \quad ( )$$

$$Q_k = Q_k(\otimes \times \Delta t) \quad ( )$$

Ⓢ

$$Q_j(\otimes \Delta t) = \frac{e^{\otimes \Delta t} - E_j(\otimes \Delta t)}{(\otimes \Delta t)^j} \quad ( )$$

$$E_j(\otimes \Delta t) = \sum_{k=0}^{j-1} \frac{(\otimes \Delta t)^k}{k!}; j = 0, 1, \dots \quad ( )$$

Ⓢ

$$\frac{\partial^2 u_y}{\partial t^2} + Q_y \times \frac{\partial u_y}{\partial t} = \frac{1}{\rho} \left( \frac{\partial}{\partial x} \left( \mu \frac{\partial u_y}{\partial x} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial u_y}{\partial z} \right) \right) + \frac{f_y}{\rho} \quad ( )$$

$$\mathbf{U} = \begin{pmatrix} u_y \\ v_y \end{pmatrix}; \mathbf{L} = \begin{pmatrix} 0 & \mathbf{I} \\ \otimes_y & -Q_y \end{pmatrix}; F = \frac{1}{\rho} \begin{pmatrix} 0 \\ f_y \end{pmatrix} \quad ( )$$

semi group

$$e^{\Delta t \mathbf{L}} = \mathbf{I} + \Delta t \mathbf{L} + \frac{\Delta t^2}{2!} \mathbf{L}^2 + \frac{\Delta t^3}{3!} \mathbf{L}^3 + \frac{\Delta t^4}{4!} \mathbf{L}^4 + \dots$$

$$\beta_0 = \mathbf{I} + \Delta t \mathbf{L} + \frac{\Delta t^2}{2!} \mathbf{L}^2 + \frac{\Delta t^3}{3!} \mathbf{L}^3 + \frac{\Delta t^4}{4!} \mathbf{L}^4 + \dots \quad ( )$$

$$\beta_0 = \mathcal{Q}_1(\mathbf{L}\Delta t) = (e^{\mathbf{L}\Delta t} - \mathcal{Q}_1(\mathbf{L}\Delta t))^{-1} \gamma = 0 \text{ \& } M = 1 \Rightarrow$$

$$\beta_0 = \mathcal{Q}_1(\mathbf{L}\Delta t) = (e^{\mathbf{L}\Delta t} - \mathcal{Q}_1(\mathbf{L}\Delta t))^{-1} \gamma = 0 \text{ \& } M = 1 \Rightarrow \quad ( )$$

:[ ]

$$\beta_0 = \mathbf{I} + \frac{\Delta t}{2} \mathbf{L} + \frac{\Delta t^2}{6} \mathbf{L}^2 + \frac{\Delta t^3}{24} \mathbf{L}^3 + \dots \quad ( )$$

[ ]

[ ]

x

x

y

SH

Qy

$$Q_y(x, z) = ax \left( e^{bx \cdot x^2} + e^{bx(x-nx)^2} \right) + az \left( e^{bz \cdot z^2} + e^{bz(z-nz)^2} \right)$$

$$ax = az = 10000; bx = bz = -0.02; nx = nz = 128 \quad ( )$$

Qy



SH ( )

- ny nx  
: y x

$\rho_{wall,found} = 1.5 \times 2.5 \text{ ton} / m^3$

$\rho = 2 \frac{\text{ton}}{m^3} \quad \mu = 1.8 \times 10^4 \text{ kPa}$

$\mu_{wall,found} = 4 \times 10^7 \text{ kPa}$   
k=128

( )

t=0.0048 ( )

( )

( )

( \* \*

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( )

( )

SH

) ( )  
 $\rho_2 = 1.5 \times 2.5 \frac{\text{ton}}{m^3}$  و  $\rho_1 = 0.5 \times 2.5 \frac{\text{ton}}{m^3}$  (

)

.[ [ ]

$\mu_2 = 100 \times 10^7 \text{ kPa}$   $\mu_1 = 4 \times 10^7 \text{ kPa}$

( )

$\mu_1 = 4 \times 10^7 \text{ kPa}$

( )

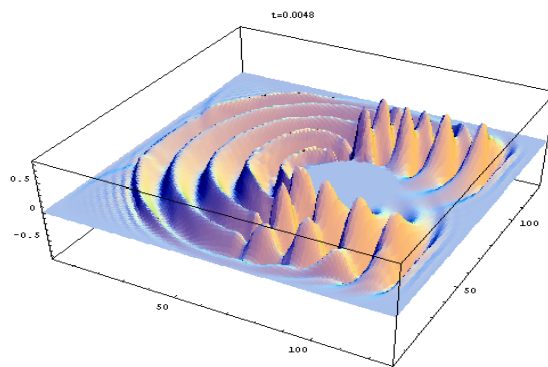
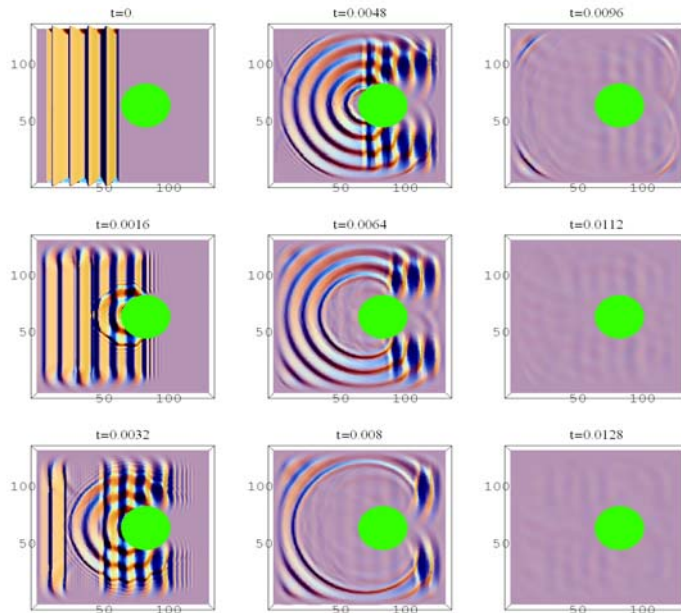
$\rho = 1.8 (\text{ton} / m^3)$

c ( )

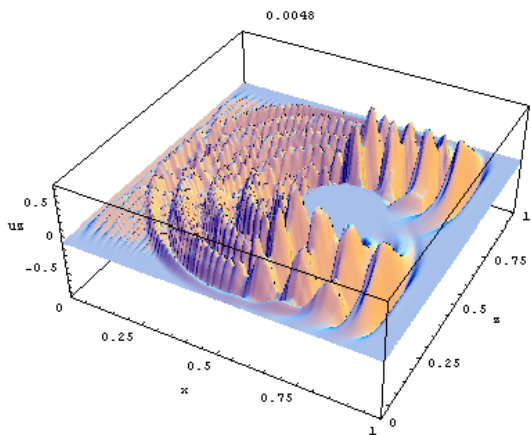
$\rho_2 = 1.5 \times 2.5 (\text{ton} / m^3)$   $\rho_1 = 0.5 \times 2.5 (\text{ton} / m^3)$

\*

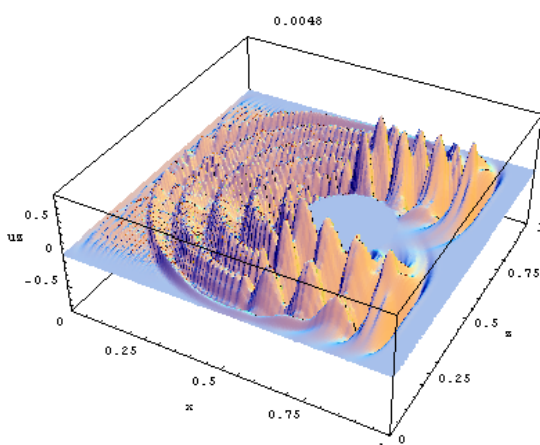
Daubechies



$n \times n = 128 \times 128$ ,  $t(\text{calculate}) = 569 \text{ sec}$

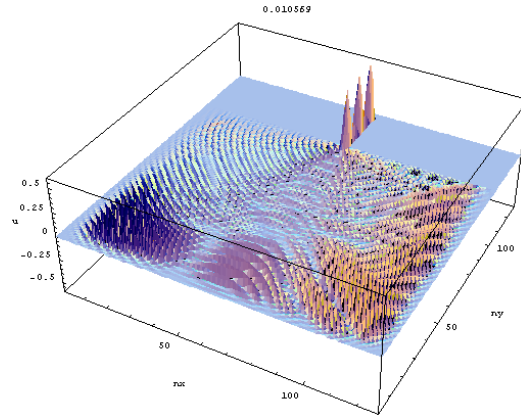
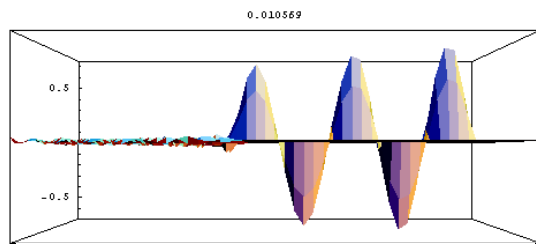
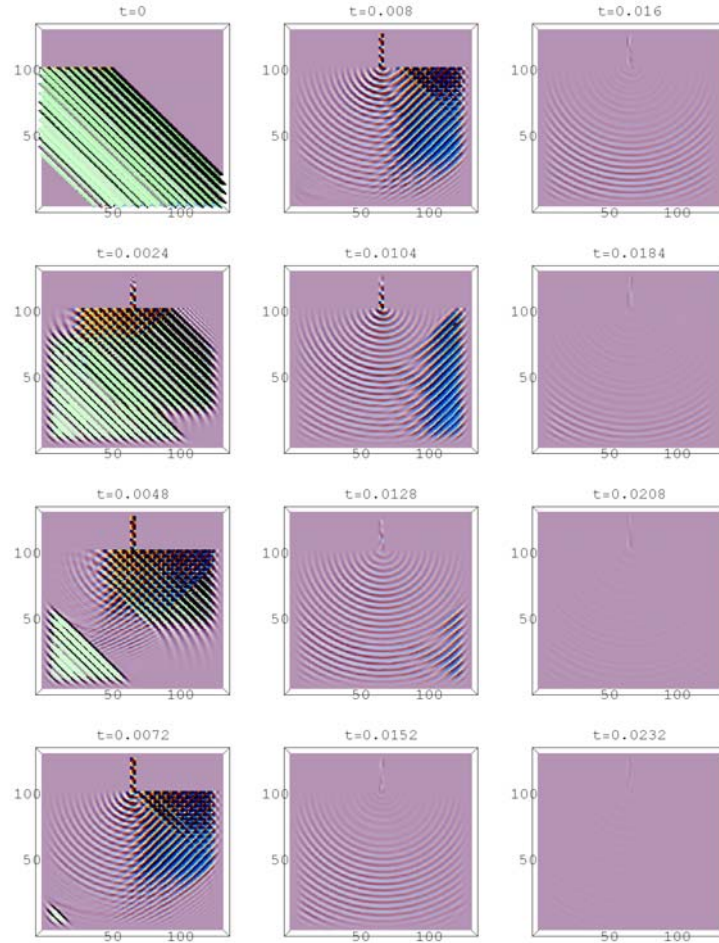


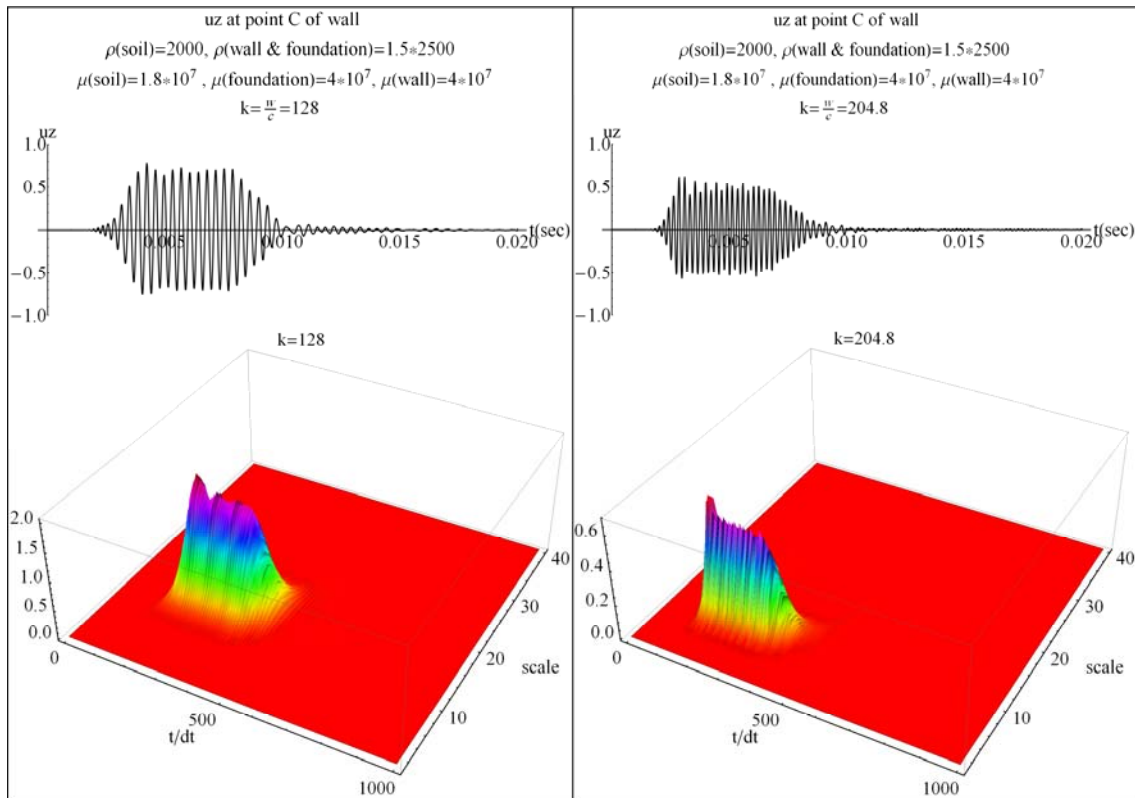
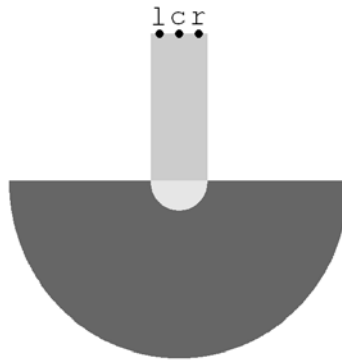
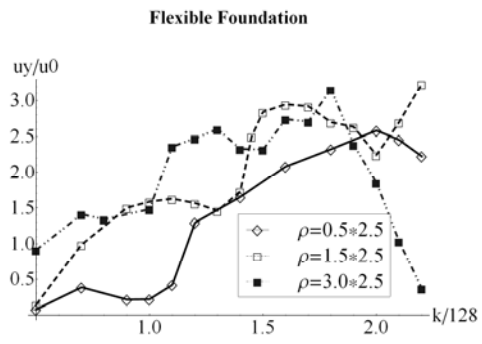
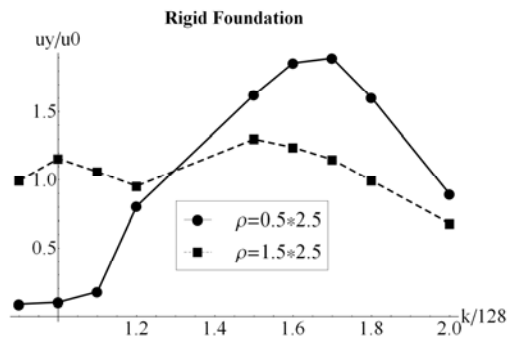
$dx(\text{max}) = 0.007$ ,  $t = 370 \text{ sec}$



$dx(\text{max}) = 0.005$ ,  $t = 932 \text{ sec}$

**dx**





$$e^{\Delta t L} \left( \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right) \left( \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right)$$

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- 1 - Multiresolution
- 2 - Resolution
- 3 - Support
- 4 - Compact
- 5 - NS-Form